MATH 54 Lecture Notes 6

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1 Subspaces

Let V be a vector space, and let W be a subset of V. Then if W is also a vector space, we say that W is a *subspace* of V. Examples are lines inside planes, planes inside 3-space, or polynomials inside the space of continuous functions.

To check whether a subset of a vector space is a subspace, we only need to check closure under addition and scalar multiplication. Note that this only works if the operations of vector addition and scalar multiplication have been left as they are in the larger vector space.

2 Null Spaces

Let A be an $m \times n$ matrix with real entries. Then the null space of A, written $NS(A)$, is the subset of $\mathbb{R}^n$

$$\{\mathbf{x} \in \mathbb{R}^n \mid A\mathbf{x} = \mathbf{0}\}.$$

If A has complex entries, then $NS(A)$ is a subset of $\mathbb{C}^n$, similarly defined.

The null space of any matrix is always a subspace of $\mathbb{R}^n$ or $\mathbb{C}^n$.