MATH 54 Lecture Notes 13

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1 Diagonalization

To diagonalize a matrix means to give a similarity transformation between it and a diagonal matrix. Let A be an $n \times n$ matrix, and suppose $A = P^{-1}BP$. Then

$$\begin{aligned}\det(\lambda I - A) &= \det(P^{-1}\lambda IP - P^{-1}BP)\\ &= \det(P^{-1}(\lambda I - B)P)\\ &= \det(P^{-1})\det(\lambda I - B)\det P\\ &= \det(\lambda I - B),\end{aligned}$$

so A and B have the same characteristic polynomial, and hence the same eigenvalues. Therefore, when we diagonalize a matrix, it will be similar to a matrix with the same eigenvalues on the diagonal.

Diagonalization is simply a change of basis to a basis of eigenvectors. A matrix is diagonalizable if and only if there is a basis of eigenvectors.

Exercise 19.

Exercise 24.

Exercise 26.

2 Spectral Theorem

Let A be a self-adjoint $n \times n$ matrix; that is, $A = A^*$. Then A is diagonalizable, and all its eigenvalues are real. Furthermore, we can come up with an orthogonal matrix that diagonalizes A, since eigenvectors with different eigenvalues are orthogonal when A is self-adjoint.

Exercise 1.

Exercise 7.