MATH 54 Lecture Notes 11

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July 19, 2006

1 Similarity of Matrices

Let $T : V \to V$ be a linear transformation on a finite-dimensional vector space V. Let B and C be two ordered bases for V. Let P be the transition matrix from C to B. Then

$$[T]_C = P^{-1}[T]_B P.$$

Any similarity transformation can be thought of as a change of basis for some linear map, since any invertible matrix is a change of basis matrix from the standard basis to the basis consisting of its columns.

Exercise 23.

Exercise 24.

Exercise 29.

2 Determinants

The determinant of a 1×1 matrix is simply the unique entry of that matrix. For an $n \times n$ matrix, the cofactor C_{ij} of the matrix is given by

$$C_{ij} = (-1)^{i+j} M_{ij},$$

where M_{ij} is the determinant of the matrix with the i-th row and j-th column removed. Then

$$\det A = \sum_{i=1}^{n} (A)_{ij} C_{ij}$$

for any j.

Computing 1×1 determinants is easy. See that this definition agrees with our previous notion of determinants for 2×2 and 3×3 matrices.

We only have the concept of a determinant for square matrices.

Some facts about determinants:

- $(\det A)(\det B) = \det(AB)$
- $\det(A^T) = \det A$
- The determinant of any lower triangular matrix or upper triangular matrix is the product of the elements on the diagonal.
- The determinant of any invertible matrix is nonzero. We can see this from the fact that the determinant of any elementary matrix is nonzero.
- Any matrix with nonzero determinant is invertible. This can be seen from the explicit formula for A^{-1}:

$$(A^{-1})_{ij} = \frac{C_{ji}}{\det A}.$$

Exercise 11.

Exercise 25. Sometimes the determinant is a polynomial.

Exercise 26. In general, if each entry of a matrix is a linear polynomial, the determinant of an $n \times n$ matrix with be a polynomial of degree n.