

1. Answer the following questions with “true” or “false.”

- (a) If  $Q$  is orthogonal, then so is  $Q^4$ .
- (b) Let  $\mathbf{v}_1$  and  $\mathbf{v}_2$  be orthogonal. Then they are also linearly independent.
- (c) Let  $A$  be an  $n \times n$  matrix such that the only eigenvectors of  $A$  are multiples of  $(1, 0, 0)^T$ . Then  $A$  is invertible.
- (d) Let  $P$  be the orthogonal projection matrix for a subspace  $V$  of  $\mathbb{R}^n$ , where  $\dim V < n$ . Then  $P$  has an eigenvalue 0.
- (e) Let  $J_1$  and  $J_2$  be two  $n \times n$  matrices in Jordan canonical form. Then  $J_1 J_2$  is also in Jordan canonical form.

2. Give matrices  $S$  and  $J$  such that  $A = SJS^{-1}$  and  $J$  is in Jordan canonical form.

$$A = \begin{pmatrix} 1/3 & 1 & -2/3 \\ -4/3 & 1 & -4/3 \\ 2/3 & 2 & 5/3 \end{pmatrix}$$

3. Give matrices  $T$  and  $L$  such that  $B = TLT^{-1}$  and  $L$  is in Jordan canonical form.

$$B = \begin{pmatrix} 4/3 & -1/6 & 1/6 \\ 1/3 & 5/6 & 1/6 \\ -1/3 & 1/6 & 5/6 \end{pmatrix}$$

4. Find the best fit parabola to the data points  $(0, 0)$ ,  $(1, 2)$ ,  $(-1, -1)$ , and  $(2, 0)$  using the least squares method.

5. Find an orthonormal basis for the subspace

$$S = \{f \in P_3 \mid f(1) = 0\}$$

of  $P_3$  with respect to the inner product

$$\langle f, g \rangle = \int_{-1}^1 f(x)\overline{g(x)} dx.$$