

1. Answer the following “true” or “false.”

(a) Define a function

$$(u_1, u_2, u_3, u_4) \cdot (v_1, v_2, v_3, v_4) = u_1 v_1 + u_2 v_3 + u_3 v_2 + u_4 v_4.$$

This is an inner product on  $\mathbb{R}^4$ .

(b) Define a function

$$(u_1, u_2, u_3, u_4) \cdot (v_1, v_2, v_3, v_4) = |u_1 v_1| + |u_2 v_2| + |u_3 v_3| + |u_4 v_4|.$$

This is an inner product on  $\mathbb{R}^4$ .

(c) Let  $A$  be a matrix with rank 0. Then  $A = 0$ .

2. Let  $B$  be the standard basis in  $P_3$ . Let  $C$  be the ordered basis  $1, (x - 1), (x - 1)^2, (x - 2)^3$ . Find a change of basis matrix from  $B$  to  $C$ , and use it to give the coordinates of  $(x - 2)^2$  with respect to  $C$ .

3. Give a  $3 \times 3$  matrix that rotates the  $xz$ -plane by 180 degrees and scales the  $y$ -axis by a factor of 2. Then give the inverse of this matrix.

4. Give a basis for  $NS(A)$ . Then give a basis for  $RS(A)$ . What is  $\dim NS(A)$ ? What is  $rk(A)$ ? As far as what  $A$  is, just draw a  $3 \times 5$  matrix and put random complex numbers in it.

5. Let  $\langle f, g \rangle = \int_{-1}^1 f(x) \overline{g(x)} dx$ .

(a) Compute  $\langle 1, x \rangle$ .

(b) Give a basis for the following subspace of  $P_3$ .

$$S = \{f \in P_3 \mid \langle f, x \rangle = 0\}$$