Jordan Canonical Form Cheat Sheet

GSI Carter

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In all cases, A is a nondiagonalizable 3×3 matrix.

- $\chi_A(\lambda) = (\lambda - \lambda_0)^2(\lambda - \lambda_1)$, where $\lambda_0 \neq \lambda_1$, and $\dim NS(A - \lambda_0 I) = 1$
  1. $J = \begin{pmatrix} \lambda_0 & 1 & \\ & \lambda_0 & \\ & & \lambda_1 \end{pmatrix}$
  2. $\mathbf{v}_3 \in NS(A - \lambda_1 I)$ such that $\mathbf{v}_3 \neq \mathbf{0}$
  3. $\mathbf{v}_2 \in NS(A - \lambda_0 I)^2$ such that $\mathbf{v}_2 \notin NS(A - \lambda_0 I)$
  4. $\mathbf{v}_1 = (A - \lambda_0 I)\mathbf{v}_2$
- $\chi_A(\lambda) = (\lambda - \lambda_0)^3$ and $\dim NS(A - \lambda_0 I) = 2$
  1. $J = \begin{pmatrix} \lambda_0 & 1 & \\ & \lambda_0 & \\ & & \lambda_0 \end{pmatrix}$
  2. $\mathbf{v}_2 \notin NS(A - \lambda_0 I)$
  3. $\mathbf{v}_1 = (A - \lambda_0 I)\mathbf{v}_2$
  4. $\mathbf{v}_3 \in NS(A - \lambda_0 I)$ such that $\mathbf{v}_3$ is not a scalar multiple of $\mathbf{v}_1$
- $\chi_A(\lambda) = (\lambda - \lambda_0)^3$ and $\dim NS(A - \lambda_0 I) = 1$
  1. $J = \begin{pmatrix} \lambda_0 & 1 & \\ & \lambda_0 & 1 \\ & & \lambda_0 \end{pmatrix}$
  2. $\mathbf{v}_3 \notin NS(A - \lambda_0 I)^2$
  3. $\mathbf{v}_2 = (A - \lambda_0 I)\mathbf{v}_3$
  4. $\mathbf{v}_1 = (A - \lambda_0 I)\mathbf{v}_2$

Then if $S = (\mathbf{v}_1 \mid \mathbf{v}_2 \mid \mathbf{v}_3)$, S is invertible and $A = SJS^{-1}$.