1. Solve the following initial value problem.

$$\begin{aligned} y'' - 4y' + 4y &= 0 \\ y'(0) &= 1 \\ y(0) &= 0 \end{aligned}$$

The characteristic equation of the differential equation is $0 = r^2 - 4r + 4 = (r-2)^2$, so the general solution is given by

$$y = c_1 e^{2t} + c_2 t e^{2t},$$

the derivative of which is

$$y' = 2c_1 e^{2t} + 2c_2 t e^{2t} + c_2 e^{2t}.$$

Then the initial conditions give us the equations

$$\begin{aligned} 0 &= c_1 \\ 1 &= 2c_1 + c_2. \end{aligned}$$

Therefore $c_1 = 0$ and $c_2 = 1$, so the solution to the initial value problem is

$$y = te^{2t}.$$

2. (a) Compute the Wronskian $W(t, t^2, t^3)$.

$$\begin{aligned} W(t, t^2, t^3) &= \begin{vmatrix} t & t^2 & t^3 \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} \\ &= t \begin{vmatrix} 2t & 3t^2 \\ 2 & 6t \end{vmatrix} - 1 \begin{vmatrix} t^2 & t^3 \\ 2 & 6t \end{vmatrix} \\ &= t(12t^2 - 6t^2) - (6t^3 - 2t^3) \\ &= 2t^3. \end{aligned}$$

(b) Are the functions t, t^2, and t^3 linearly independent?

Yes, since they are polynomials of different degrees. Also, the Wronskian is nonzero for all $t \neq 0$.