1. Answer the following statements "true" or "false."

   (a) Let A and B be 3×3 matrices. Then $\det(AB) = \det(BA)$.
   True, since $\det(AB) = (\det A)(\det B) = (\det B)(\det A) = \det(BA)$.

   (b) Let A and B be 3×3 orthogonal matrices. Then AB is orthogonal.
   True. A matrix being orthogonal is equivalent to its inverse and transpose being equal. Then since $(AB)^T = B^T A^T = B^{-1}A^{-1} = (AB)^{-1}$, the matrix AB is orthogonal.

2. Find an orthogonal basis for the subspace $S = \{p \in P_3 \mid p'(1) = 0\}$ of P_3 with respect to the inner product $\langle p, q\rangle = p(0)q(0) + p(1)q(1) + p(2)q(2)$. (Note that this is not an inner product on P_3, but it is an inner product on S.)

   Since the condition for a polynomial to be in S is centered around the point $x = 1$, let's start with the basis $\{1, (x-1), (x-1)^2, (x-1)^3\}$ of P_3 and then try to find a basis for the subspace. Let $p(x) = a + b(x-1) + c(x-1)^2 + d(x-1)^3$. Then $p'(x) = b + 2c(x-1) + 3d(x-1)^2$, so $p'(1) = 0$ if and only if $b = 0$. Therefore $f_1 = 1$, $f_2 = (x-1)^2$, and $f_3 = (x-1)^3$ form a basis for S. Now we only need to perform Gram-Schmidt.

$$
\begin{aligned}
p_1 &= 1 \\
p_2 &= (x-1)^2 - \frac{\langle (x-1)^2, 1\rangle}{\langle 1, 1\rangle} = (x-1)^2 - \frac{2}{3} \\
p_3 &= (x-1)^3 - \frac{\langle (x-1)^3, 1\rangle}{\langle 1, 1\rangle} - \frac{\left\langle (x-1)^3, (x-1)^2 - \frac{2}{3}\right\rangle}{\left\langle (x-1)^2 - \frac{2}{3}, (x-1)^2 - \frac{2}{3}\right\rangle}\left((x-1)^2 - \frac{2}{3}\right) = (x-1)^3
\end{aligned}
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