

1. Answer the following statements “true” or “false.”

- (a) The function $f : M_{33} \rightarrow \mathbb{R}$ given by $f(A) = \det A$ is a linear transformation.
False, since $f(I) = 1$ but $f(2I) = 8 \neq 2f(I)$.
- (b) The function $f : P_2 \rightarrow P_1$ given by $f(a_0 + a_1x + a_2x^2) = a_2x + 3$ is a linear transformation.
False, since $f(0) \neq 0$.
- (c) Let $\mathbf{u} \in \mathbb{R}^4$. Then the function $f : \mathbb{R}^4 \rightarrow \mathbb{R}$ given by $f(\mathbf{v}) = \mathbf{u} \cdot \mathbf{v}$ is a linear transformation.
True.

2. Let $B = \{1, x, x^2\}$ and $C = \{1 + x^2, 1 - x^2, x - x^2\}$. Give the transition matrix from B to C (that is, the matrix P such that $P[q]_B = [q]_C$ for all $q \in P_2$).

Since

$$\begin{aligned}1 &= \frac{1}{2}(1 + x^2) + \frac{1}{2}(1 - x^2) + 0 \cdot (x - x^2), \\x &= \frac{1}{2}(1 + x^2) - \frac{1}{2}(1 - x^2) + (x - x^2), \\x^2 &= \frac{1}{2}(1 + x^2) - \frac{1}{2}(1 - x^2) + 0 \cdot (x - x^2),\end{aligned}$$

we have that

$$P = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 0 & 2 & 0 \end{pmatrix}.$$