

1. Answer the following questions “true” or “false.”

(a) The set  $R = \{f \in C[-1, 1] \mid f(0) = 1\}$  is a subspace of  $C[-1, 1]$ .

False. The set  $R$  does not contain the zero function.

(b) The vectors  $(2, -1, 3)$  and  $(2, -1, -3)$  are linearly independent in  $\mathbb{R}^3$ .

True. The only way for two vectors to be linearly dependent is for one to be a scalar multiple of the other, which is not the case here.

(c) The set  $S = \text{Span}\{(2, -1, 3), (2, -1, -3)\}$  is a subspace of  $\mathbb{R}^3$ .

True. The span of any set of vectors is always a subspace.

2. Let  $\mathbf{v}_1 = (1, 1, -1)$  and  $\mathbf{v}_2 = (-1, -3, 2)$ . Use Gram-Schmidt to find vectors  $\mathbf{p}_1$  and  $\mathbf{p}_2$  such that  $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\} = \text{Span}\{\mathbf{p}_1, \mathbf{p}_2\}$  and  $\mathbf{p}_1 \cdot \mathbf{p}_2 = 0$ .

First, we let  $\mathbf{p}_1 = \mathbf{v}_1 = (1, 1, -1)$ . Then we let

$$\begin{aligned}\mathbf{p}_2 &= \mathbf{v}_2 - \frac{\mathbf{v}_2 \cdot \mathbf{p}_1}{\mathbf{p}_1 \cdot \mathbf{p}_1} \mathbf{p}_1 \\ &= (-1, -3, 2) - \frac{-6}{3}(1, 1, -1) \\ &= (-1, -3, 2) + 2(1, 1, -1) \\ &= (1, -1, 0).\end{aligned}$$