1. For each of the following matrices, either compute the inverse or state that no inverse exists.

(a) $A = \begin{pmatrix} 1 & 3 & 1 \\ -1 & -4 & 0 \\ 0 & 2 & -2 \end{pmatrix}$

The augmented matrix $(A \mid I)$ row reduces as follows:

$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ -1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 2 & -2 & 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 2 & -2 & 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 2 & 2 & 1 \end{pmatrix}$$

Therefore A is not invertible.

(b) $B = \begin{pmatrix} 1 & 3 & 1 \\ -1 & -4 & 0 \\ 0 & 2 & 2 \end{pmatrix}$

The augmented matrix $(B \mid I)$ row reduces as follows:

$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ -1 & -4 & 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 2 & 2 & 0 & 0 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 4 & 2 & 2 & 1 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & 1/4 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & 1/4 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & -1/2 & 1/4 \\ 0 & 0 & 1 & 1/2 & 1/2 & 1/4 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 3 & 0 & 1/2 & -1/2 & 1/4 \\ 0 & 1 & 0 & -1/2 & -1/2 & 1/4 \\ 0 & 0 & 1 & 1/2 & 1/2 & 1/4 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 0 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & -1/2 & -1/2 & 1/4 \\ 0 & 0 & 1 & 1/2 & 1/2 & 1/4 \end{pmatrix}$$

Therefore $B^{-1} = \begin{pmatrix} 2 & 1 & -1 \\ -1/2 & -1/2 & 1/4 \\ 1/2 & 1/2 & 1/4 \end{pmatrix}$.

2. Answer the following with either "true" or "false": for any invertible 2×2 invertible matrices A and B, AB is invertible and

$$(AB)^{-1} = A^{-1}B^{-1}.$$

False. Any invertible matrices A and B such that $AB \neq BA$ can serve as a counterexample. For instance, consider $A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.