

1. Answer the following questions “true” or “false.”

(a) The following system of equations has at least one solution.

$$\begin{aligned} 3x + 4y &= 0 \\ \pi x - y &= 0 \\ -7x + 19y &= 0 \end{aligned}$$

True, since it is a homogeneous system.

(b) Let $A = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$. Then $AB = BA$ for all 2×2 matrices B .

False. The matrix $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ serves as a counterexample.

(c) Let $A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. Then $AB = BA$ for all 2×2 matrices B .

True, since $A = -I$.

2. Give all solutions to the following system of equations.

$$\begin{aligned} 2w - x - 2y - 3z &= 1 \\ 3x + 9y + 12z &= -8 \\ 4w + x + 5y - 3z &= 0 \end{aligned}$$

This system gives the augmented matrix

$$\begin{pmatrix} 2 & -1 & -2 & -3 & 1 \\ 0 & 3 & 9 & 12 & -8 \\ 4 & 1 & 5 & -3 & 0 \end{pmatrix}.$$

This matrix row reduces as follows:

$$\begin{aligned} &\begin{pmatrix} 2 & -1 & -2 & -3 & 1 \\ 0 & 3 & 9 & 12 & -8 \\ 0 & 3 & 9 & 3 & -2 \end{pmatrix} \\ &\begin{pmatrix} 2 & -1 & -2 & -3 & 1 \\ 0 & 3 & 9 & 12 & -8 \\ 0 & 0 & 0 & -9 & 6 \end{pmatrix} \end{aligned}$$

Therefore y is a free variable, $z = -\frac{2}{3}$,

$$x = \frac{-8 - 9y - 12z}{3} = -4z - \frac{8}{3} - 3y = -3y,$$

and

$$w = \frac{1 + x + 2y + 3z}{2} = \frac{1}{2} - \frac{3}{2}y + y - 1 = -\frac{1}{2}y - \frac{1}{2}.$$