

1. For each of the following statements, write the word “true” or “false.”

(a) The initial value problem $y' = \frac{2}{3}y^3$, $y(0) = 0$ has a unique solution on the interval $(-1, 1)$.
There is a mistake in this problem.

(b) The initial value problem $ty'' - 2y' + y = 0$, $y(1) = 1$, $y'(1) = 0$ has a unique solution on the interval $(0, \infty)$.
True. See the theorems of 3.2.

2. Solve the following initial value problem:

$$\begin{aligned}y'' - 2y' - 3y &= 0 \\y(0) &= 1 \\y'(0) &= 0\end{aligned}$$

The characteristic equation is given by $0 = r^2 - 2r - 3 = (r - 3)(r + 1)$. Therefore the general solution of the differential equation is $y = c_1e^{3t} + c_2e^{-t}$, and $y' = 3c_1e^{3t} - c_2e^{-t}$. Then the initial conditions give us $1 = c_1 + c_2$ and $0 = 3c_1 - c_2$, so that $c_1 = \frac{1}{4}$ and therefore $c_2 = \frac{3}{4}$. The solution to the initial value problem is then given by

$$y = \frac{1}{4}e^{3t} + \frac{3}{4}e^{-t}.$$