1. Let A and B be any $n \times n$ matrices with an eigenvalue of 2. For each of the following statements, write the word "true" or "false."
   (a) The matrix A^2 must have an eigenvalue of 4.
   True. See exercise 23 in section 5.2.
   (b) The matrix AB must have an eigenvalue of 4.
   False. Consider $A = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

2. Find any nonzero eigenvector for the following matrix and its associated eigenvalue:

$$\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & 4 \\ -1 & 2 & 1 \end{pmatrix}$$

Let A denote the given matrix. Then the characteristic polynomial of A is

$$\begin{aligned} \det(\lambda I - A) &= \begin{vmatrix} \lambda - 1 & 0 & -3 \\ -2 & \lambda - 1 & -4 \\ 1 & -2 & \lambda - 1 \end{vmatrix} \\ &= (\lambda - 1)\left[(\lambda - 1)^2 - 8\right] - 3\left[4 - (\lambda - 1)\right] \\ &= (\lambda - 1)(\lambda^2 - 2\lambda - 7) + 3(\lambda - 5) \\ &= \lambda^3 - 2\lambda^2 - 7\lambda - \lambda^2 + 2\lambda + 7 + 3\lambda - 15 \\ &= \lambda^3 - 3\lambda^2 - 2\lambda - 8 \\ &= (\lambda - 4)(\lambda^2 + \lambda + 2). \end{aligned}$$

Therefore $\lambda = 4$ is an eigenvalue. To find an eigenvector, we need to compute the null space of

$$4I - A = \begin{pmatrix} 3 & 0 & -3 \\ -2 & 3 & -4 \\ 1 & -2 & 3 \end{pmatrix}.$$

This row reduces to

$$\begin{pmatrix} 1 & -2 & 3 \\ 0 & -1 & 2 \\ 0 & 6 & -12 \end{pmatrix}$$

and then to

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{pmatrix}.$$

Therefore the vector $(1, 2, 1)$ is an eigenvector for A with eigenvalue 4.