1. For each of the following statements, write the word "true" or "false."

   (a) Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformations. Then the function S from U to W defined by $S(\mathbf{u}) = T_2(T_1(\mathbf{u}))$ for any $\mathbf{u} \in U$ is a linear transformation.
   True, since $T_2(T_1(\mathbf{u}+\mathbf{v})) = T_2(T_1(\mathbf{u}) + T_1(\mathbf{v})) = T_2(T_1(\mathbf{u})) + T_2(T_1(\mathbf{v}))$ and $T_2(T_1(\alpha\mathbf{u})) = T_2(\alpha T_1(\mathbf{u})) = \alpha T_2(T_1(\mathbf{u}))$.

   (b) The function $E : C[a,b] \to \mathbb{R}$ given by $E(f) = \int_a^b x \cdot f(x)\, dx$ for any $f \in C[a,b]$ is a linear transformation.
   True, since

$$\begin{aligned} E(f+g) &= \int_a^b x[f(x)+g(x)]\, dx \\ &= \int_a^b [x \cdot f(x) + x \cdot g(x)]\, dx \\ &= \int_a^b x \cdot f(x)\, dx + \int_a^b x \cdot g(x)\, dx \\ &= E(f) + E(g), \end{aligned}$$

   and

$$E(\alpha f) = \int_a^b x[\alpha f(x)]\, dx = \alpha \int_a^b x \cdot f(x)\, dx = \alpha E(f).$$

2. Give the coordinates of the polynomial $p(x) = 1 - x + x^3$ in terms of the basis $\{1, x-1, (x-1)^2, (x-1)^3\}$ of the vector space P_3. Write your answer in the form of a column vector.

   We have to find scalars a, b, c, and d such that

$$a + b(x-1) + c(x^2 - 2x + 1) + d(x^3 - 3x^2 + 3x - 1) = 1 - x + x^3.$$

   This can be done by solving the system

$$\begin{pmatrix} 1 & -1 & 1 & -1 & 1 \\ 0 & 1 & -2 & 3 & -1 \\ 0 & 0 & 1 & -3 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

   which row reduces to

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

   Therefore, the coordinate vector is:

$$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}.$$