

- Let V be the real vector space of all polynomials of degree 7 or less. Let W be the set of all polynomials p in V such that $p(4) = 0$, which is a subspace of V . For each of the following statements, write the word “true” or “false.”
 - Any basis for W can be extended to a basis for V .

True. This is true about subspaces in general, by a theorem in 3.6.
 - Any basis for V has some subset which is a basis for W .

False. For a counterexample, consider the basis $\{1, x, x^2, x^3, x^4, x^5, x^6, x^7\}$ of V , none of whose elements are in W .
- Find a basis for the null space of the following matrix.

$$\begin{pmatrix} 2 & 3 & 2 & 2 \\ 4 & 3 & 3 & 4 \end{pmatrix}$$

The matrix row reduces as follows:

$$\begin{pmatrix} 2 & 3 & 2 & 2 \\ 0 & -3 & -1 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & -3 & -1 & 0 \end{pmatrix}$$
$$\begin{pmatrix} 2 & 0 & 1 & 2 \\ 0 & 3 & 1 & 0 \end{pmatrix}$$

Therefore the null space consists of vector of the form $(-\frac{a}{2} - b, -\frac{a}{3}, a, b)$, so a basis of the null space is given by $\{(-\frac{1}{2}, -\frac{1}{3}, 1, 0), (-1, 0, 0, 1)\}$.