1. For each of the following statements, write the word "true" or "false."

   (a) The set of matrices of the form

   $$\begin{pmatrix} a & a-b \\ a+b & b \end{pmatrix}$$

   is a linear vector space.

   TRUE. The operations of addition and scalar multiplication have not been modified, so we only need to check that the given set is closed under those operations. We have that

   $$\lambda \begin{pmatrix} a & a-b \\ a+b & b \end{pmatrix} = \begin{pmatrix} \lambda a & \lambda a - \lambda b \\ \lambda a + \lambda b & \lambda b \end{pmatrix}$$

   and

   $$\begin{pmatrix} a & a-b \\ a+b & b \end{pmatrix} + \begin{pmatrix} c & c-d \\ c+d & d \end{pmatrix} = \begin{pmatrix} a+c & (a+c)-(b+d) \\ (a+c)+(b+d) & b+d \end{pmatrix},$$

   so the given set of matrices is a linear vector space.

   (b) The set of matrices of the form

   $$\begin{pmatrix} a & ab \\ ab & b \end{pmatrix}$$

   is a linear vector space.

   FALSE. The set is not closed under scalar multiplication, as demonstrated by

   $$2\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}.$$

2. Let $\mathbf{v}_1 = (1,2,0)$ and $\mathbf{v}_2 = (-1,3,1)$. Find perpendicular vectors $\mathbf{p}_1$ and $\mathbf{p}_2$ such that $\text{Span}\{\mathbf{p}_1, \mathbf{p}_2\} = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

   We use the Gram-Schmidt process. Let $\mathbf{p}_1 = \mathbf{v}_1$ and

   $$\begin{aligned} \mathbf{p}_2 &= \mathbf{v}_2 - \frac{\mathbf{v}_2 \cdot \mathbf{p}_1}{\mathbf{p}_1 \cdot \mathbf{p}_1}\mathbf{p}_1 \\ &= (-1,3,1) - \frac{5}{5}(1,2,0) \\ &= (-2,1,1). \end{aligned}$$