For each of the following matrices, either compute its inverse or state that there is no inverse. In both matrices, a is a non-zero constant.

1.

$$\begin{pmatrix} 1 & a & 0 & 0 \\ 0 & 1 & -a & 0 \\ 0 & 0 & 1 & a \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

By row reduction, we get

$$\begin{pmatrix} 1 & a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -a & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & a & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -a & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & a & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & a & -a^2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 1 & -a & -a^2 & a^3 \\ 0 & 1 & 0 & 0 & 0 & 1 & a & -a^2 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

So, the inverse is

$$\begin{pmatrix} 1 & -a & -a^2 & a^3 \\ 0 & 1 & a & -a^2 \\ 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2.

$$\begin{pmatrix} a & 4 & 3 \\ 0 & 2 & 1 \\ -a & -2 & -2 \end{pmatrix}$$

The bottom row is the difference of the top two rows. Therefore, this matrix has a row of all zeroes when it is row reduced, and it is not invertible.