

1. For each of the following statements, write the word “true” or “false.”

(a) $xy = k$ is a linear equation, where x and y are variables and k is a constant.

False, because the variables are being multiplied together.

(b) The following system of equations has at least one solution.

$$x + y - z = 0$$

$$3x + 2y + z = 0$$

$$x - 2y - 2z = 0$$

$$7x - y + 8z = 0$$

True, because every homogeneous system has at least the trivial solution.

2. Find all solutions to the following system of equations.

$$x + y + z = 5$$

$$2x - y + z = 10$$

$$4x + y + 3z = 20$$

The augmented matrix associated to this system can be row reduced as follows:

$$\begin{pmatrix} 1 & 1 & 1 & 5 \\ 2 & -1 & 1 & 10 \\ 4 & 1 & 3 & 20 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 5 \\ 0 & -3 & -1 & 0 \\ 0 & -3 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 1 & 5 \\ 0 & 1 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore the choice of z is arbitrary, and we have that $y = -\frac{z}{3}$ and $x = 5 - y - z = 5 - \frac{2z}{3}$.