Quiz 12 Solutions

1. Let $\mathbf{F}(x,y,z) = \left\langle 4yz, -xz^2, \sin(xyz)\right\rangle$. Compute the flux of curl $\mathbf{F}$ across the the part of the paraboloid $z = x^2 + y^2$ lying below the plane $z = 4$, oriented upward.

   By Stokes' Theorem, we can compute the line integral of $\mathbf{F}$ along the boundary of the surface instead. This boundary can be parametrized by $\mathbf{r}(t) = \langle 2\cos t, 2\sin t, 4\rangle$, where $t \in [0, 2\pi]$. Then $\mathbf{r}'(t) = \langle -2\sin t, 2\cos t, 0\rangle$ and $\mathbf{F}(\mathbf{r}(t)) = \langle 32\sin t, -32\cos t, \sin(16\sin t\cos t)\rangle$. Thus, the line integral is

$$\begin{aligned}\int_C \mathbf{F}\cdot d\mathbf{r} &= \int_0^{2\pi} \mathbf{F}(\mathbf{r}(t))\cdot \mathbf{r}'(t)\,dt\\ &= -64\int_0^{2\pi} \cos^2 t + \sin^2 t\,dt\\ &= -128\pi.\end{aligned}$$

2. Find the flux of the vector field $\mathbf{F}(x,y,z) = \langle e^{\cos x}, \sin(xy), z(\sin x)e^{\cos x} - xz\cos(xy)\rangle$ through the surface $x^2 + y^2 + z^2 = 9$, oriented outward.

   By the Divergence Theorem, we can instead compute the integral of div $\mathbf{F}$ on the interior of the surface. Since

$$\operatorname{div}\mathbf{F} = -(\sin x)e^{\cos x} + x\cos(xy) + (\sin x)e^{\cos x} - x\cos(xy) = 0,$$

   the flux integral is 0.