Quiz 5 Solutions

1. Find the tangent plane for $z = e^{\sin^2 x + \cos^2 y - 1}$ at the point $(\pi, \pi, 1)$.
   $\frac{\partial z}{\partial x} = 2z \sin x \cos x$ and $\frac{\partial z}{\partial y} = -2z \sin y \cos y$. So, the tangent plane is given by $z - 1 = 0 \cdot (x - \pi) + 0 \cdot (x - \pi) = 0$, or $z = 1$.

2. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ in terms of s and t only, where $z = e^{\alpha} \sin \beta$, $\alpha = s^2 + t^2$, and $\beta = s - 2t$.

$$\begin{aligned}\frac{\partial z}{\partial s} &= \frac{\partial \alpha}{\partial s}\frac{\partial z}{\partial \alpha} + \frac{\partial \beta}{\partial s}\frac{\partial z}{\partial \beta} \\ &= (2s)(e^{\alpha} \sin \beta) + (1)(e^{\alpha} \cos \beta) \\ &= e^{s^2+t^2}(2s \sin(s - 2t) + \cos(s - 2t)).\end{aligned}$$

$$\begin{aligned}\frac{\partial z}{\partial t} &= \frac{\partial \alpha}{\partial t}\frac{\partial z}{\partial \alpha} + \frac{\partial \beta}{\partial t}\frac{\partial z}{\partial \beta} \\ &= (2t)(e^{\alpha} \sin \beta) + (-2)(e^{\alpha} \cos \beta) \\ &= 2e^{s^2+t^2}(t \sin(s - 2t) - \cos(s - 2t)).\end{aligned}$$