Quiz 4 Solutions

1. Find the unit tangent vector $\mathbf{T}(t)$ to the curve $\mathbf{r}(t) = (\sin t)\mathbf{i} + 4\mathbf{j} - (\tan t)\mathbf{k}$ at $t = \frac{\pi}{4}$.

   The tangent vector is $\mathbf{r}'(t) = (\cos t)\mathbf{i} - \left(\frac{1}{\cos^2 t}\right)\mathbf{k}$. At $t = \frac{\pi}{4}$, $\mathbf{r}'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\mathbf{i} - 2\mathbf{k}$. So, $\left|\mathbf{r}'\left(\frac{\pi}{4}\right)\right| = \sqrt{\frac{1}{2} + 4} = \frac{3}{\sqrt{2}}$. Thus, $\mathbf{T}\left(\frac{\pi}{4}\right) = \frac{\mathbf{r}'\left(\frac{\pi}{4}\right)}{\left|\mathbf{r}'\left(\frac{\pi}{4}\right)\right|} = \frac{1}{3}\mathbf{i} - \frac{2\sqrt{2}}{3}\mathbf{k}$.

2. Find the limit, if it exists, or show that the limit does not exist:

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz + zx}{x^2 + y^2 + z^2}$$

   Suppose we approach the origin along the positive z-axis. Then $x = y = 0$, and the limit of the given function is 0. Then suppose we approach along the line in the xz-plane where $x = z$. Then $y = 0$, and the given function simplifies to $\frac{x^2}{2x^2}$, which approaches $\frac{1}{2}$ as x goes to 0.
   Since $0 \neq \frac{1}{2}$, the limit does not exist.