1. (a)

$$\begin{aligned}
\iint_D (x-2y)\, dA &= \int_1^3 \int_{1+x}^{2x} (x-2y)\, dy\, dx \\
&= \int_1^3 \left. (xy-y^2)\right|_{y=1+x}^{2x}\, dx \\
&= \int_1^3 2x^2 - 4x^2 - x - x^2 + 1 + 2x + x^2\, dx \\
&= \int_1^3 1 + x - 2x^2\, dx \\
&= \left. \left(x + \frac{x^2}{2} - \frac{2x^3}{3} \right) \right|_1^3 \\
&= 3 + \frac{9}{2} - 18 - 1 - \frac{1}{2} + \frac{2}{3} \\
&= -\frac{34}{3}.
\end{aligned}$$

(b)

$$\begin{aligned}
\int_C f\, ds &= \int_0^1 \frac{t^2}{3} \sqrt{t^4 + \frac{4t^2}{9}}\, dt \\
&= \frac{1}{9} \int_0^1 t^3 \sqrt{9t^2+4}\, dt \\
&= \frac{1}{9} \cdot \frac{1}{18} \cdot \frac{1}{9} \int_4^{13} (u-4)\sqrt{u}\, du \qquad \text{(where } u = 9t^2+4\text{)} \\
&= \frac{1}{9} \cdot \frac{1}{18} \cdot \frac{1}{9} \left. \left(\frac{2}{5} u^{5/2} - \frac{8}{3} u^{3/2} \right) \right|_4^{13}
\end{aligned}$$

(c)

$$\begin{aligned}
\iiint_E e^x\, dV &= \int_0^1 \int_x^1 \int_0^{x+y} e^x\, dz\, dy\, dx \\
&= \int_0^1 \int_x^1 (x+y)e^x\, dy\, dx \\
&= \int_0^1 (1-x)xe^x + \frac{(1-x^2)e^x}{2}\, dx
\end{aligned}$$

2. (a) A hyperboloid of one sheet is $x^2+y^2-z^2=1$. A hyperboloid of two sheets is $x^2-y^2-z^2=1$.

   (b) The given equation is equivalent to $(x-3)^2+(y+1)^2-(z+1)^2=0$. This is a cone.

   (c) To find the point of intersection, we must solve the equation $e^{2t}-e^{-2t}=0$. The only solution is at $t=0$. Let $F(x,y,z)=(x-3)^2+(y+1)^2-(z+1)^2$, so that our surface is given by the equation $F(x,y,z)=0$. Then the normal vector to the surface at any point is given by

$$\nabla F(x,y,z) = \langle 2(x-3), 2(y+1), -2(z+1) \rangle.$$

   The point of intersection is $(3+1/\sqrt{2}, 1/\sqrt{2}-1, 0)$, so the normal vector is $\langle \sqrt{2}, \sqrt{2}, -2 \rangle$. The tangent vector to the curve is $\langle e^t/\sqrt{2}, e^t/\sqrt{2}, -e^{-t} \rangle$, which is $\langle 1/\sqrt{2}, 1/\sqrt{2}, -1 \rangle$ at $t=0$. Since these two vectors are scalar multiples of one another, the curve is normal to the surface at the point of intersection.

3. (a) See the book.
   (b) Done in the review session.
   (c) Straightforward.
4. (a) The Cartesian coordinates of a point (ρ, θ, ϕ) are given by $x = \rho \cos\theta \sin\phi$, $y = \rho \sin\theta \sin\phi$, and $z = \rho \cos\phi$.
   (b) In this situation, we can parametrize the surface as

$$\mathbf{r}(\theta, \phi) = \langle f(\theta)\cos\theta \sin\phi, f(\theta)\sin\theta\sin\phi, f(\theta)\cos\phi\rangle$$

with $(\theta, \phi) \in D$. Therefore

$$\mathbf{r}_\theta(\theta,\phi) = \langle (f'(\theta)\cos\theta - f(\theta)\sin\theta)\sin\phi, (f'(\theta)\sin\theta + f(\theta)\cos\theta)\sin\phi, f'(\theta)\cos\phi\rangle .$$

Similarly,

$$\mathbf{r}_\phi(\theta,\phi) = \langle f(\theta)\cos\theta\cos\phi, f(\theta)\cos\theta\cos\phi, -f(\theta)\sin\phi\rangle .$$

Then we have

$$\begin{aligned}
\mathbf{r}_\theta \times \mathbf{r}_\phi &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (f'(\theta)\cos\theta - f(\theta)\sin\theta)\sin\phi & (f'(\theta)\sin\theta + f(\theta)\cos\theta)\sin\phi & f'(\theta)\cos\phi \\ f(\theta)\cos\theta\cos\phi & f(\theta)\sin\theta\cos\phi & -f(\theta)\sin\phi \end{vmatrix} \\
&= -f(\theta)[(f'(\theta)\sin\theta + f(\theta)\cos\theta)\sin^2\phi + f'(\theta)\sin\theta\cos^2\phi]\mathbf{i} \\
&\quad + f(\theta)[f'(\theta)\cos\theta\cos^2\phi + (f'(\theta)\cos\theta - f(\theta)\sin\theta)\sin^2\phi]\mathbf{j} \\
&\quad + f(\theta)\sin\phi\cos\phi[(f'(\theta)\cos\theta - f(\theta)\sin\theta)\sin\theta - (f'(\theta)\sin\theta + f(\theta)\cos\theta)\cos\theta]\mathbf{k} \\
&= -f(\theta)[f'(\theta)\sin\theta + f(\theta)\cos\theta\sin^2\phi]\mathbf{i} \\
&\quad + f(\theta)[f'(\theta)\cos\theta - f(\theta)\sin\theta\sin^2\phi]\mathbf{j} - [f(\theta)]^2\sin\phi\cos\phi\mathbf{k}.
\end{aligned}$$

Therefore

$$\begin{aligned}
|\mathbf{r}_\theta \times \mathbf{r}_\phi| &= \sqrt{[f(\theta)]^4\sin^2\phi\cos^2\phi + [f(\theta)]^2(f'(\theta)^2 + f(\theta)^2\sin^4\phi)} \\
&= \sqrt{f(\theta)^4\sin^2\phi + f(\theta)^2 f'(\theta)^2}.
\end{aligned}$$

Then the area of S is given by

$$\iint_S dS = \iint_D |\mathbf{r}_\theta \times \mathbf{r}_\phi|\, dA = \iint_D \sqrt{f(\theta)^4\sin^2\phi + f(\theta)^2 f'(\theta)^2}\, d\theta\, d\phi.$$

5. (a) The surface can be parametrized by $x = \cos^2\theta\sin\phi$, $y = \sin\theta\cos\theta\sin\phi$, and $z = \cos\theta\cos\phi$.
   The intersection of the surface with the xy-plane is given by setting $z = 0$, which is equivalent to either $\cos\theta = 0$ or $\cos\phi = 0$. $\cos\theta = 0$ gives only the single point $(0,0,0)$, while $\cos\phi = 0$ means that $\sin\phi = 1$, since $0 \le \phi \le \pi/2$. Therefore, the intersection is the curve parametrized by $x = \cos^2\theta$ and $y = \sin\theta\cos\theta$ for $0 \le \theta \le \pi/2$ (since this curve already includes the origin).
   For the intersection with the xz-plane, we set $y = 0$, which is the same as saying $\sin\theta = 0$, $\cos\theta = 0$, or $\sin\phi = 0$. Setting $\cos\theta = 0$ gives only the point $(0,0,0)$. Setting $\sin\theta = 0$ means that $\cos\theta = 1$, since $0 \le \theta \le \pi/2$. Then the intersection includes the curve parametrized by $x = \sin\phi$ and $z = \cos\phi$ for $0 \le \phi \le \pi/2$. Setting $\sin\phi = 0$ means that $\cos\phi = 1$, so that the intersection also includes the curve parametrized by $x = 0$ and $z = \cos\theta$.
   For the intersection with the yz-plane, we set $x = 0$, which is the same as saying $\cos\theta = 0$ or $\sin\phi = 0$. Setting $\cos\theta = 0$ gives only the point $(0,0,0)$. Setting $\sin\phi = 0$ means that $\cos\phi = 1$. Then the intersection consists of the curve parametrized by $y = 0$ and $z = \cos\theta$ for $0 \le \theta \le \pi/2$.

(b) By the Divergence Theorem, this is the same as computing the flux of the vector field $\mathbf{F}(x, y, z) = \langle x, 0, 0 \rangle$ across the surface. So,

$$\begin{aligned}
\iint_S \mathbf{F} \cdot d\mathbf{S} &= \iint_D \mathbf{F} \cdot (\mathbf{r}_\theta \times \mathbf{r}_\phi)\, dA \\
&= \int_0^{\pi/2} \int_0^{\pi/2} -\cos^3\theta \sin\phi[\cos^2\theta \sin^2\phi - \sin^2\theta]\, d\theta\, d\phi \\
&= \int_0^{\pi/2} \sin^2\theta \cos^3\theta\, d\theta \int_0^{\pi/2} \sin\phi\, d\phi - \int_0^{\pi/2} \cos^5\theta\, d\theta \int_0^{\pi/2} \sin^3\phi\, d\phi \\
&= \frac{2}{15} - \frac{8}{15} \cdot \frac{2}{3} \\
&= -\frac{2}{9}.
\end{aligned}$$

Therefore the volume of the solid is $\frac{2}{9}$.

6. (a) Look it up in the index. The theorem is the whole paragraph, not just the red box.
   (b) Done this a few times already.
   (c) Blah blah. Verify the conditions of the Implicit Function Theorem, and you're done.

7. (a) Let E, and let S be its boundary, oriented outward. Let $\mathbf{F}$ be a vector field defined on E whose components have continuous partial derivatives. Then

$$\iiint_E \operatorname{div} \mathbf{F}\, dV = \iint_S \mathbf{F} \cdot d\mathbf{S}.$$

   (b) I did this this morning.
   (c) The divergence of a constant vector field is 0. Done!