

Practice Midterm 2 Solutions

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1. Let θ be the angle between \mathbf{a} and \mathbf{b} . Then

$$\cos^2 \theta = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \right)^2 = \frac{1}{4}.$$

So, $|\sin \theta| = \sqrt{1 - \cos^2 \theta} = \frac{\sqrt{3}}{2}$. So,

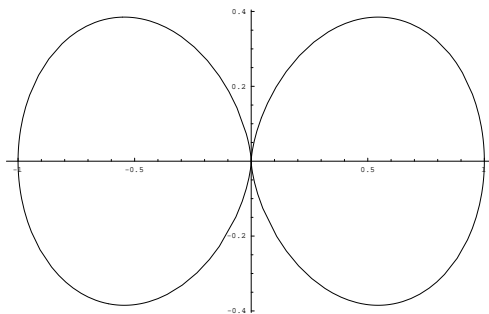
$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \cdot |\sin \theta| = 6\sqrt{3}.$$

- 2.

$$\begin{aligned} \frac{(2 \cosh t)^2}{8} + \left(\frac{1}{\sqrt{2}} \cosh t \right)^2 - \frac{(3 \sinh t)^2}{9} &= \frac{\cosh^2 t}{2} + \frac{\cosh^2 t}{2} - \sinh^2 t \\ &= 1. \end{aligned}$$

Since every point on the curve satisfies the relation defining the quadric, the curve lies on the quadric. The quadric is a one-sheeted hyperboloid.

3. The graph is given below.



The enclosed area is given by

$$\begin{aligned} 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} r^2 d\theta &= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^4 \theta d\theta \\ &= \left(\frac{3\theta}{8} + \frac{\sin(2\theta)}{4} + \frac{\sin(4\theta)}{32} \right) \Big|_{\theta=-\frac{\pi}{2}}^{\frac{\pi}{2}} \\ &= \frac{3\pi}{8}. \end{aligned}$$