Practice Midterm 1 Solutions

February 10, 2005

1. Let θ be the angle between $\mathbf{a}$ and $\mathbf{b}$. Then

$$\cos^2 \theta = \left(\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}||\mathbf{b}|} \right)^2 = \frac{1}{2}.$$

So, $|\sin \theta| = \sqrt{1 - \cos^2 \theta} = \frac{\sqrt{2}}{2}$. The area of the parallelogram spanned by $\mathbf{a}$ and $\mathbf{b}$ is equal to $|\mathbf{a} \times \mathbf{b}|$, and

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \cdot |\sin \theta| = 3\sqrt{2}.$$

So, the area of the parallelogram is $3\sqrt{2}$.

2. Differentiating, we obtain the velocity $\mathbf{v}(t) = \langle 2t + 40, 6t \rangle$ and acceleration $\mathbf{a}(t) = \langle 2, 6 \rangle$. The velocity and acceleration being perpendicular is equivalent to the condition $\mathbf{v}(t) \cdot \mathbf{a}(t) = 0$, and

$$\mathbf{v}(t) \cdot \mathbf{a}(t) = \langle 2t + 40, 6t \rangle \cdot \langle 2, 6 \rangle = 40t + 80.$$

So, the particle has perpendicular velocity and acceleration if and only if $t = -2$.

3. Since $\cosh^2 t - \sinh^2 t = 1$ for all $t \in \mathbb{R}$, $x^2 - y^2 = 1$. So, the graph of the parametric curve is contained within the hyperbola $x^2 - y^2 = 1$. However, $\cosh t$ and $\sinh t$ are both continuous, so we can get at most one piece of the hyperbola. Since $\cosh t \geq 1$ for all t, we can only get the right piece. Since $\lim_{t \to \infty} \sinh t = \infty$ and $\lim_{t \to -\infty} \sinh t = -\infty$, we get the entire right piece.