1. Let $A = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}$. Compute e^{At}.

   The characteristic polynomial of A is

   $$\begin{vmatrix} 2-\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2.$$

   Therefore $(A-I)^2 = 0$, so

   $$e^{At} = e^t e^{(A-I)t} = e^t(I + (A-I)t) = e^t \begin{pmatrix} 1+t & -t \\ t & 1-t \end{pmatrix}.$$

2. Let $B = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$. Compute e^{Bt}.

   By inspection, we see that the characteristic polynomial of B is $(\lambda-2)(\lambda-1)$ and that the eigenspace for the eigenvalue 2 is spanned by $\begin{pmatrix} 1 & 0 \end{pmatrix}^T$. The eigenspace for the eigenvalue 1 is

   $$NS(B-I) = NS\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} = \operatorname{span}\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}.$$

   The a fundamental matrix for B is given by

   $$\mathbf{X}(t) = \begin{pmatrix} e^{2t} & -e^t \\ 0 & e^t \end{pmatrix}.$$

   Then

   $$\begin{aligned} e^{Bt} &= \mathbf{X}(t)[\mathbf{X}(0)]^{-1} \\ &= \begin{pmatrix} e^{2t} & -e^t \\ 0 & e^t \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}^{-1} \\ &= \begin{pmatrix} e^{2t} & -e^t \\ 0 & e^t \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \\ &= \begin{pmatrix} e^{2t} & e^{2t} - e^t \\ 0 & e^t \end{pmatrix}. \end{aligned}$$