Find the general solution to each of the following differential equations.

1. $y''' - 3y'' + y' - 3y = 0$

   The characteristic equation is

   $$r^3 - 3r^2 + r - 3 = (r-3)(r^2+1) = 0.$$

   Therefore the general solution is

   $$y = c_1 e^{3t} + c_2 \cos t + c_3 \sin t.$$

2. $y''' - 3y'' + y' - 3y = e^{3t}$

   We will look for a particular solution $y_p = ate^{3t}$. We have

   $$\begin{aligned} y_p' &= ae^{3t}(3t+1) \\ y_p'' &= ae^{3t}(9t+6) \\ y_p''' &= ae^{3t}(27t+27). \end{aligned}$$

   Then we have that $10a = 1$, so $y_p = \frac{1}{10}te^{3t}$. Therefore the general solution is

   $$y = \frac{1}{10}te^{3t} + c_1 e^{3t} + c_2 \cos t + c_3 \sin t.$$