Math 54, Quiz 10 Solutions Section 114 GSI Carter

Solve the following initial value problems.

1.

$$\begin{aligned} y'' + y &= \cos t \\ y(0) &= 0 \\ y'(0) &= 1 \end{aligned}$$

The homogeneous solution is $y_h = c_1 \cos t + c_2 \sin t$, so our guess for a particular solution will be $y_p = t(a \cos t + b \sin t)$. Then $y_p' = a \cos t + b \sin t + t(b \cos t - a \sin t)$ and

$$y_p'' = b \cos t - a \sin t + b \cos t - a \sin t - t(a \cos t + b \sin t),$$

so that $y_p'' + y_p = 2b \cos t - 2a \sin t$, and therefore $b = \frac{1}{2}$ and $a = 0$. Then the general solution is

$$y = c_1 \cos t + c_2 \sin t + \frac{t \sin t}{2},$$

and the initial conditions give us

$$\begin{aligned} 0 &= c_1 \\ 1 &= c_2. \end{aligned}$$

Therefore the solution to the initial value problem is

$$y = \sin t + \frac{t \sin t}{2}.$$

2.

$$\begin{aligned} y'' - y' - 2y &= 1 \\ y(0) &= -\frac{1}{2} \\ y'(0) &= 1 \end{aligned}$$

The homogeneous solution is $y_h = c_1 e^{2t} + c_2 e^{-t}$, and a particular solution is given by $y_p = -\frac{1}{2}$. Therefore the general solution is

$$y = c_1 e^{2t} + c_2 e^{-t} - \frac{1}{2},$$

and the initial conditions give us

$$\begin{aligned} 0 &= c_1 + c_2 \\ 1 &= 2c_1 - c_2. \end{aligned}$$

Therefore $c_1 = \frac{1}{3}$ and $c_2 = -\frac{1}{3}$, so the solution to the initial value problem is

$$y = \frac{1}{3} e^{2t} - \frac{1}{3} e^{-t} - \frac{1}{2}.$$