Solve the following initial value problems.

1.

$$
\begin{aligned}
y'' + y' &= 3 \\
y(0) &= 0 \\
y'(0) &= 1
\end{aligned}
$$

The homogeneous solution is $y_h = c_1 + c_2 t$. Therefore our guess for a particular solution will be $y_p = at^2$. Then $2a = 3$, so $a = \frac{2}{3}$. Then $y = c_1 + c_2 t + \frac{2}{3}t^2$ is the general solution, and the initial values give us

$$
\begin{aligned}
0 &= c_1 \\
1 &= c_2.
\end{aligned}
$$

Therefore the solution to the initial value problem is $y = t + \frac{2}{3}t^2$.

2.

$$
\begin{aligned}
y'' - y' - 6y &= 1 \\
y(0) &= -\frac{1}{6} \\
y'(0) &= 1
\end{aligned}
$$

The homogeneous solution is $y_h = c_1 e^{3t} + c_2 e^{-2t}$, and a particular solution is given by $y_p = -\frac{1}{6}$. Therefore the general solution is $y = c_1 e^{3t} + c_2 e^{-2t} - \frac{1}{6}$, so the initial conditions give

$$
\begin{aligned}
0 &= c_1 + c_2 \\
1 &= 3c_1 - 2c_2.
\end{aligned}
$$

Therefore $c_1 = \frac{1}{5}$ and $c_2 = -\frac{1}{5}$, so the solution to the initial value problem is

$$
y = \frac{1}{5}e^{3t} - \frac{1}{5}e^{-2t} - \frac{1}{6}.
$$