1. Find an orthogonal matrix Q and an upper triangular matrix R such that $A = QR$.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

We must apply Gram-Schmidt to the columns of A, in order. This gives us

$$\mathbf{v}_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

and

$$\mathbf{v}_2 = \sqrt{2} \begin{pmatrix} 1/2 \\ -1/2 \end{pmatrix} = \sqrt{2} \left[-\frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right].$$

Then

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \sqrt{2}\mathbf{v}_1$$

and

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}(\mathbf{v}_1 + \mathbf{v}_2).$$

Therefore

$$Q = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

and

$$R = \begin{pmatrix} \sqrt{2} & 1/\sqrt{2} \\ 0 & 1/\sqrt{2} \end{pmatrix}.$$

2. Compute the best fit line through the points $(0,0)$, $(1,1)$, and $(3,5)$ in the least-squares sense.
   We need to solve the equation $A^T A\mathbf{x} = A^T\mathbf{b}$, where

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 3 \end{pmatrix}$$

and

$$\mathbf{b} = \begin{pmatrix} 0 \\ 1 \\ 5 \end{pmatrix}$$

Then

$$A^T A = \begin{pmatrix} 3 & 4 \\ 4 & 10 \end{pmatrix}$$

and

$$A^T\mathbf{b} = \begin{pmatrix} 6 \\ 16 \end{pmatrix},$$

so that

$$\mathbf{x} = \frac{1}{14}\begin{pmatrix} 10 & -4 \\ -4 & 3 \end{pmatrix}\begin{pmatrix} 6 \\ 16 \end{pmatrix} = \begin{pmatrix} -2/7 \\ 12/7 \end{pmatrix}.$$

Therefore the best fit line is $y = \frac{12}{7}x - \frac{2}{7}$.