1. Find matrices C and P such that $A = PCP^{-1}$ and C is of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, where $a, b \in \mathbb{R}$.

$A = \begin{pmatrix} -3 & 2 \\ -4 & 1 \end{pmatrix}$

First, we need to compute the characteristic polynomial of A.

$$\det(A - \lambda I) = \begin{vmatrix} -3-\lambda & 2 \\ -4 & 1-\lambda \end{vmatrix} = \lambda^2 + 2\lambda + 5$$

Therefore the eigenvalues of A are $-1 \pm 2i$. The eigenspace for the eigenvalue $-1 + 2i$ is

$$NS(A - (-1+2i)I) = NS \begin{pmatrix} -2-2i & 2 \\ -4 & 2-2i \end{pmatrix}$$

for which a basis is given by the single vector $\begin{pmatrix} 2 \\ 2+2i \end{pmatrix}$. Therefore we can take

$$P = \begin{pmatrix} 2 & 0 \\ 2 & 2 \end{pmatrix}$$

and

$$C = \begin{pmatrix} -1 & 2 \\ -2 & -1 \end{pmatrix}.$$

2. Compute the orthogonal projection of the vector $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ onto the line between the origin and $\begin{pmatrix} 4 \\ 3 \end{pmatrix}$.

$$\frac{15}{16+9} \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 12/5 \\ 9/5 \end{pmatrix}$$