Find all eigenvalues for each of the following matrices. For each eigenvalue, find a basis for the corresponding eigenspace.

1. $A = \begin{pmatrix} 2 & 4 \\ -1 & -2 \end{pmatrix}$

   The characteristic polynomial of $A$ is

   $$\det(A - \lambda I) = \begin{vmatrix} 2-\lambda & 4 \\ -1 & -2-\lambda \end{vmatrix} = \lambda^2,$$

   so the only eigenvalue is 0. The eigenspace is the null space of $A$, for which a basis consists of the single vector $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

2. $B = \begin{pmatrix} 7 & -12 \\ 4 & -7 \end{pmatrix}$

   The characteristic polynomial of $B$ is

   $$\det(B - \lambda I) = \begin{vmatrix} 7-\lambda & -12 \\ 4 & -7-\lambda \end{vmatrix} = \lambda^2 - 49 + 48 = (\lambda+1)(\lambda-1),$$

   so the eigenvalues of $B$ are 1 and $-1$. The eigenspace for the eigenvalue 1 is the null space of the matrix

   $$B - I = \begin{pmatrix} 6 & -12 \\ 4 & -8 \end{pmatrix}$$

   for which a basis is given by the single vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$. The eigenspace for $-1$ is the null space of the matrix

   $$B + I = \begin{pmatrix} 8 & -12 \\ 4 & -6 \end{pmatrix}$$

   for which a basis is given by the single vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.