

Find all eigenvalues for each of the following matrices. For each eigenvalue, find a basis for the corresponding eigenspace.

1. $A = \begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}$

The characteristic polynomial of A is

$$\det(A - \lambda I) = \begin{vmatrix} 2 - \lambda & -2 \\ -2 & 2 - \lambda \end{vmatrix} = \lambda^2 - 4\lambda.$$

Therefore the eigenvalues are 0 and 4. The eigenspace for 0 is the null space of A , for which a basis is given by the single vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. The eigenspace for 4 is the null space of the matrix

$$A - 4I = \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$$

for which a basis is given by the single vector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

2. $B = \begin{pmatrix} 3 & -1 \\ 4 & -1 \end{pmatrix}$

The characteristic polynomial of B is

$$\det(B - \lambda I) = \begin{vmatrix} 3 - \lambda & -1 \\ 4 & -1 - \lambda \end{vmatrix} = \lambda^2 - 2\lambda + 1 = (\lambda - 1)^2.$$

Therefore the only eigenvalue of B is 1. The eigenspace is the null space of the matrix

$$B - I = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix}$$

for which a basis is given by the single vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.