

Consider the following two subsets of \mathbb{P}_3 . One of them is a subspace of \mathbb{P}_3 , while the other is not. For each subset, indicate whether or not it is a subspace. For the one that is a subspace, give a basis for it and its dimension.

1. $V = \{f \in \mathbb{P}_3 \mid f(0) = f(1) = 0\}$

This is a subspace, and a basis for it is $\{x^3 - x^2, x^2 - x\}$. Therefore its dimension is 2.

2. $W = \{f \in \mathbb{P}_3 \mid f(0) \cdot f(1) = 0\}$

This is not a subspace. It is not closed under addition.