1. Find all solutions to the matrix equation $A\mathbf{x} = \mathbf{0}$, where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

The matrix row reduces as follows:

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

Therefore the set of all solutions to the equation $A\mathbf{x} = \mathbf{0}$ is the span of the single vector

$$\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}.$$

2. Write down a 4×3 matrix whose columns are linearly independent.

There are many possible correct answers, such as the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$