

1. Find all solutions to the matrix equation $A\mathbf{x} = \mathbf{0}$, where

$$A = \begin{pmatrix} 1 & -1 & 5 & 4 \\ 2 & -1 & 3 & -1 \\ -2 & 0 & 6 & 1 \end{pmatrix}.$$

The matrix A row reduces as follows:

$$\begin{pmatrix} 1 & -1 & 5 & 4 \\ 0 & 1 & -7 & -9 \\ 0 & -2 & 16 & 9 \end{pmatrix}$$
$$\begin{pmatrix} 1 & -1 & 5 & 4 \\ 0 & 1 & -7 & -9 \\ 0 & 0 & 2 & -9 \end{pmatrix}$$

Therefore the set of solutions is all scalar multiples of the vector

$$\begin{pmatrix} 28 \\ 81 \\ 9 \\ 2 \end{pmatrix}$$

2. Write down a 3×4 matrix whose columns span \mathbb{R}^3 .

There are many correct answers here. One example is the following matrix:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$