

1. Find all solutions to the following system of equations. Please explicitly indicate any free variables.

$$\begin{aligned} 3x - y + 4z &= 1 \\ 4y + 2z &= -4 \\ x - y + z &= 1 \end{aligned}$$

The augmented matrix for the system row reduces as follows:

$$\begin{aligned} &\begin{pmatrix} 3 & -1 & 4 & 1 \\ 0 & 4 & 2 & -4 \\ 1 & -1 & 1 & 1 \end{pmatrix} \\ &\begin{pmatrix} 0 & 2 & 1 & -2 \\ 0 & 4 & 2 & -4 \\ 1 & -1 & 1 & 1 \end{pmatrix} \\ &\begin{pmatrix} 0 & 2 & 1 & -2 \\ 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 1 \end{pmatrix} \end{aligned}$$

Therefore z is a free variable, $y = -1 - \frac{z}{2}$, and

$$x = 1 + y - z = -\frac{3z}{2}.$$

2. Find all solutions to the following system of equations. Please explicitly indicate any free variables.

$$\begin{aligned} 3x - y + 2z &= 0 \\ 4y + 4z &= 8 \\ x - y + z &= 1 \end{aligned}$$

The augmented matrix for the system row reduces as follows:

$$\begin{aligned} &\begin{pmatrix} 3 & -1 & 2 & 0 \\ 0 & 4 & 4 & 8 \\ 1 & -1 & 1 & 1 \end{pmatrix} \\ &\begin{pmatrix} 0 & 2 & -1 & -3 \\ 0 & 4 & 4 & 8 \\ 1 & -1 & 1 & 1 \end{pmatrix} \\ &\begin{pmatrix} 0 & 2 & -1 & -3 \\ 0 & 0 & 6 & 14 \\ 1 & -1 & 1 & 1 \end{pmatrix} \end{aligned}$$

Therefore $z = \frac{7}{3}$, $y = -\frac{3}{2} + \frac{z}{2} = -\frac{2}{6} = -\frac{1}{3}$, and $x = 1 + y - z = -\frac{5}{3}$.