

1. Find all solutions to the following system of equations. Please explicitly indicate any free variables.

$$\begin{aligned}y + z &= 4 \\2x + 3y &= 1 \\x + y - z &= 0\end{aligned}$$

The augmented matrix row reduces as follows:

$$\begin{pmatrix} 0 & 1 & 1 & 4 \\ 2 & 3 & 0 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 1 & 4 \\ 0 & 0 & 1 & -3 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

Therefore the unique solution to the system is  $z = -3$ ,  $y = 4 - z = 7$ , and  $x = z - y = -10$ .

2. Find all solutions to the following system of equations. Please explicitly indicate any free variables.

$$\begin{aligned}y + 3z &= 2 \\2x + 3y + z &= 2 \\x + y - z &= 0\end{aligned}$$

The augmented matrix row reduces as follows:

$$\begin{pmatrix} 0 & 1 & 3 & 2 \\ 2 & 3 & 1 & 2 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 3 & 2 \\ 0 & 1 & 3 & 2 \\ 1 & 1 & -1 & 0 \end{pmatrix}$$

Therefore  $z$  is a free variable,  $y = 2 - 3z$ , and  $x = z - y = -2 + 4z$ .