

1. Either solve the following boundary value problem or else show that it has no solution.

$$\begin{aligned}y'' + y &= 0 \\y(0) &= 0 \\y'(\pi) &= 1\end{aligned}$$

The general solution of the differential equation is $y = c_1 \cos t + c_2 \sin t$. From the boundary condition $y(0) = 0$, we get that $c_1 = 0$. Since $y' = c_2 \cos t$, we get from $y'(\pi) = 1$ that $-c_2 = 1$. Therefore this boundary value has a unique solution, $y = -\sin t$.

2. Find the Fourier series for the function $f(x) = x$ defined on the interval $[-\pi, \pi)$. You may assume that f is periodic with period 2π .

Since f is an odd function, $a_n = 0$ for all n , and

$$\begin{aligned}b_n &= \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx \\&= \frac{2}{\pi} \int_0^\pi x \sin(nx) dx \\&= \frac{2}{\pi} \left[-\frac{x \cos(nx)}{n} + \int \frac{\cos(nx)}{n} dx \right] \Big|_{x=0}^\pi \\&= \frac{2}{\pi} \left[-\frac{x \cos(nx)}{n} + \frac{\sin(nx)}{n^2} \right] \Big|_{x=0}^\pi \\&= \frac{2(-1)^{n+1}}{n}.\end{aligned}$$

Therefore the Fourier series for f is

$$2 \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin(nx).$$