1. (a) Let J be the 5×5 matrix where every entry is 1. Then the (i,j)-th entry of J^2 is

$$\sum_{k=1}^{5} 1 \cdot 1 = 5.$$

   Therefore $J^2 = 5J$.

   (b) Since $M = J - I$, $J = M + I$.

   (c) Since $J^2 = 5J$, $(M+I)^2 = 5(M+I)$. Therefore,

$$5M + 5I = M^2 + IM + MI + I^2 = M^2 + 2M + I.$$

   This gives us that $M^2 - 3M - 4I = 0$.

   (d) Since $4I = M^2 - 3M = M(M - 3I)$, $M^{-1} = \frac{1}{4}(M - 3I)$.

2. First we will show that (a) implies (b). Suppose X is such that $(BA)X = 0$. We wish to show that $X = 0$. Since B is invertible,

$$AX = B^{-1}(BA)X = B^{-1}0 = 0.$$

   Since we are assuming (a), this implies that $X = 0$.

   Now we will show that (b) implies (a), so suppose X is such that $AX = 0$. Again, we wish to show that $X = 0$. Then

$$(BA)X = B(AX) = B(0) = 0.$$

   Since we are assuming (b), this implies that $X = 0$.

3. Suppose both A and $A + B$ are invertible. Since $(I + BA^{-1})A = A + B$, $I + BA^{-1} = (A+B)A^{-1}$. Then $I + BA^{-1}$ is the product of two invertible matrices, and is therefore invertible.

   Suppose both A and $I + BA^{-1}$ are invertible. Then $A + B = (I + BA^{-1})A$ is the product of two invertible matrices, and is therefore invertible.