- If the series $\sum a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.
- The Integral Test: Suppose f is continuous on $[1, \infty)$, and positive and decreasing on $[n_0, \infty)$ for some n_0. Then let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the integral $\int_1^{\infty} f(x)\, dx$ converges.
- The p-test: The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if $p > 1$.
- The Comparison Test: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms, and let n_0 be any positive integer. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all $n \geq n_0$, then $\sum a_n$ is also convergent. If $\sum b_n$ is divergent and $a_n \geq b_n$ for all $n \geq n_0$, then $\sum a_n$ is also divergent.
- The Limit Comparison Test: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$ where c is a finite number and $c > 0$, then either both series converge or both diverge.
- The Alternating Series Test: Let b_n be a positive, decreasing sequence such that $\lim_{n\to\infty} b_n = 0$. Then the series $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.
- The Ratio Test: For a series $\sum_{n=1}^{\infty} a_n$, compute $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$. If the limit does not exist or is equal to 1, the test is inconclusive. If the limit is less than 1, the series converges absolutely. If the limit is greater than one (or infinite), then the series diverges.
- The Root Test: For a series $\sum_{n=1}^{\infty} a_n$, compute $\lim_{n\to\infty} \sqrt[n]{|a_n|}$. If the limit does not exist or is equal to 1, the test is inconclusive. If the limit is less than 1, the series converges absolutely. If the limit is greater than one (or infinite), then the series diverges.
- A power series centered at a is a series of the form $\sum_{n=0}^{\infty} c_n (x-a)^n$.
- If f has a power series representation at a, then

  $$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}.$$

  Note that $f^{(0)}(a) = f(a)$.
- Standard power series expansions:
  - $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
  - $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
  - $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$
  - $\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$
  - $\arctan x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$
  - $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^k$, where $\binom{k}{n} = \frac{k(k-1)(k-2)\cdots(k-n+1)}{n!}$ if $n \geq 1$ and $\binom{k}{0} = 1$