

- $\lim_{n \rightarrow \infty} \left(1 + \frac{r}{n}\right)^n = e^r$
- Squeeze Theorem: If $a_n \leq b_n \leq c_n$ for $n \geq n_0$ and $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} c_n = L$, then $\lim_{n \rightarrow \infty} b_n = L$.
- $\lim_{n \rightarrow \infty} |a_n| = 0$ if and only if $\lim_{n \rightarrow \infty} a_n = 0$.
- If a sequence is bounded and monotonic, then it is convergent.
- If the series $\sum a_n$ is convergent, then $\lim_{n \rightarrow \infty} a_n = 0$.
- The Integral Test: Suppose f is continuous on $[1, \infty)$, and positive and decreasing on $[n_0, \infty)$ for some n_0 . Then let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ converges if and only if the integral $\int_1^{\infty} f(x) dx$ converges.
- The p -test: The p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if $p > 1$.
- The Comparison Test: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms, and let n_0 be any positive integer. If $\sum b_n$ is convergent and $a_n \leq b_n$ for all $n \geq n_0$, then $\sum a_n$ is also convergent. If $\sum b_n$ is divergent and $a_n \geq b_n$ for all $n \geq n_0$, then $\sum a_n$ is also divergent.
- The Limit Comparison Test: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = c$ where c is a finite number and $c > 0$, then either both series converge or both diverge.

If you are trying to see whether some series converges or diverges and you get either 0 or ∞ as the limit, pick a different series to use for the comparison.

- The Alternating Series Test: Let b_n be a positive, decreasing sequence such that $\lim_{n \rightarrow \infty} b_n = 0$. Then the series $\sum_{n=1}^{\infty} (-1)^n b_n$ converges.
- A series $\sum a_n$ is called absolutely convergent if the series $\sum |a_n|$ is convergent. A series that is convergent but not absolutely convergent is called conditionally convergent.
- Any absolutely convergent series is convergent.
- The Ratio Test: For a series $\sum_{n=1}^{\infty} a_n$, compute $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$. If the limit does not exist or is equal to 1, the test is inconclusive. If the limit is less than 1, the series converges absolutely. If the limit is greater than one (or infinite), then the series diverges.
- The Root Test: For a series $\sum_{n=1}^{\infty} a_n$, compute $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$. If the limit does not exist or is equal to 1, the test is inconclusive. If the limit is less than 1, the series converges absolutely. If the limit is greater than one (or infinite), then the series diverges.
- A power series centered at a is a series of the form $\sum_{n=0}^{\infty} c_n (x - a)^n$.

- A power series can be differentiated or integrated term-by-term inside its interval of convergence.
- If f has a power series representation at a , then

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}.$$

Note that $f^{(0)}(a) = f(a)$.

- Standard power series expansions:

$$\begin{aligned} - \frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n \\ - e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} \\ - \sin x &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \\ - \cos x &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \\ - \arctan x &= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1} \\ - (1+x)^k &= \sum_{n=0}^{\infty} \binom{k}{n} x^n, \quad \text{where } \binom{k}{n} = \frac{k(k-1)(k-2)\cdots(k-n+1)}{n!} \text{ if } n \geq 1 \text{ and } \binom{k}{0} = 1 \end{aligned}$$