1 Quiz 5

1.1 8:00-9:30

1. (3pts) Determine (and justify!) whether the sequence is convergent and divergent, and give the limit if it converges.

$$a_n = \frac{(1+n)\,(1+n^2)}{\cos(n) + n^3}$$

2. (3pts) Determine whether the series $\sum_{n=1}^{\infty} ne^{-n^2}$ is convergent or divergent.

3. (4pts) Determine whether the statement true or false. If false give a counterexample.

(a) (2pts) The sequence $\{a_n\}$ diverges, the sequence $\{b_n\}$ has the limt zero then $\{a_n b_n\}$ diverges.

(b) (2pts) The sequence a_n converges. The sequence b_n is monotonically increasing and $b_n \leq a_n$. Then the sequence $\{b_n\}$ converges.

1.2 9:30-11:00

1. (3pts) Determine (and justify!) whether the sequence is convergent and divergent, and give the limit if it converges.

$$a_n = \frac{5+n^2}{(\ln n + n^2)n}$$

2. (3pts)Determine whether the series $\sum_{n=1}^{\infty} \frac{\ln n}{n+5}$ is convergent or divergent.

3. (4pts) Determine whether the statement true or false. If false give a counterexample.

(a) (2pts) Every bounded sequence converges.

(b) (2pts) The sequences $\{a_n\}$ and $\{a_n + b_n\}$ converge, then the sequence $\{b_n\}$ converges.

1.3 11:00-12:30

1. (3pts) Determine (and justify!) whether the sequence is convergent and divergent, and give the limit if it converges.

$$a_n = \frac{1}{n + \cos(n^2 + n)}$$

2. (3pts)Determine whether the series $\sum_{n=1}^{\infty} \frac{\cos(\frac{1}{n})}{n}$ is convergent or divergent.

3. (4pts) Determine whether the statement true or false. If false give a counterexample.
   (a) (2pts) Every bounded convergent sequence is monotone.

   (b) (2pts) The series $\sum_n a_n$ diverges, then $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| > 1$.

1.4 12:30-2:00

1. (3pts) Determine (and justify!) whether the sequence is convergent and divergent, and give the limit if it converges.

$$a_n = \frac{n + \cos(\frac{1}{n})}{n\cos(\pi n) + \sin(\frac{1}{n})}$$

2. (3pts)Determine whether the series $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ is convergent or divergent.

3. (4 pts.) Determine whether the statement true or false. If false give a counterexample.
   (a) (2pts) $\lim_{n\to\infty} a_n = 1$, then $\lim_{n\to\infty} a_{n+100} = 1$

   (b) (2pts) $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

1.5 2:00-3:30

1. (3pts)Determine (and justify!) whether the sequence is convergent and divergent, and give the limit if it converges.

$$a_n = \sqrt{n}(\ln(n+1) - \ln n)$$

3. (3pts)Determine whether the series $\sum_{n=1}^{\infty} \frac{n + \ln n}{n + \ln^2 n}$ is convergent or divergent.

3. (4pts) Determine whether the statement true or false. If false give a counterexample.

(a) (2pts) $a_n, b_n > 0$ and the sequence $\{a_n + b_n\}$ converges then the sequences $\{a_n\}$ and $\{b_n\}$ converge.

(b) (2pts) $a_n \to 0$, then $\sum_{n=1}^{\infty} a_n^2$ converges.

1.6 3:30-5:00

1. (3pts)Determine (and justify!) whether the sequence is convergent and divergent, and give the limit if it converges.

$$\cos(\pi n(n+1))(1 + \frac{\ln n}{n})$$

2. (3pts)Determine whether the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ is convergent or divergent.

3. (4pts) Determine whether the statement true or false. If false give a counterexample.

(a) (2pts) The sequences $\{a_n\}$ and $\{b_n\}$ converge, and $a_n < b_n$ for each n. Then $\lim_{n\to\infty} a_n < \lim_{n\to\infty} b_n$

(b) (2pts) The series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ converge, and $a_n < b_n$ for each n, then $\sum_{n=1}^{\infty}(b_n - a_n) > 0$