

1. (3pts) Give the general solution to the following differential equation:

$$y' = \frac{y-1}{\tan x}$$

$$\begin{aligned} \frac{dy}{y-1} &= \frac{\cos x}{\sin x} dx \\ \ln|y-1| &= \ln|\sin x| + C \\ y-1 &= A \sin x \\ y &= A \sin x + 1 \end{aligned}$$

2. (4pts) Find the solution to the following initial value problem:

$$\begin{aligned} \frac{y'}{\ln(x^2)} &= xy \\ y(1) &= e^3 \end{aligned}$$

$$\begin{aligned} \frac{dy}{y} &= x \ln(x^2) dx \\ \ln|y| &= \int x \ln(x^2) dx \\ &= \frac{1}{2} \int \ln u du \quad (\text{with } u = x^2) \\ &= \frac{1}{2}(u \ln u - u) + C \\ &= \frac{1}{2}(2x^2 \ln x - x^2) + C \\ y &= Ae^{x^2 \ln x - \frac{x^2}{2}} = \frac{Ax^{x^2}}{e^{\frac{x^2}{2}}} \end{aligned}$$

Since  $y(1) = e^3$ , we have that  $e^3 = \frac{A}{e^{\frac{1}{2}}}$ , or  $A = e^{7/2}$ . So,

$$y = \frac{e^{7/2} x^{x^2}}{e^{x^2/2}}.$$

3. (3pts) Match each differential equation with its direction field. (No need to show your work.)

- (a)  $y' = xy^2$  corresponds to picture number: 1  
 (b)  $y' = x - 3$  corresponds to picture number: 2  
 (c)  $y' = x - y$  corresponds to picture number: 3