1. (3pts) Find the power series representing the function $\frac{x}{1-x^3}$.

   Since $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, $\frac{1}{1-x^3} = \sum_{n=0}^{\infty} x^{3n}$. Therefore $\frac{x}{1-x^3} = \sum_{n=0}^{\infty} x^{3n+1}$.

2. (3pts) Find the Taylor series of the function $\sin x$ centered at $a = \frac{\pi}{3}$. ($\cos\frac{\pi}{3} = \frac{1}{2}, \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$)

   If $f(x) = \sin x$, then $f'(x) = \cos x$, $f''(x) = -\sin x$, and $f'''(x) = -\cos x$. If we let c_n be the sequence defined by $c_0 = \frac{\sqrt{3}}{2}$, $c_1 = \frac{1}{2}$, $c_2 = -\frac{\sqrt{3}}{2}$, $c_3 = -\frac{1}{2}$, and $c_n = c_{n-4}$ for $n \geq 4$, then the Taylor series for $f(x)$ centered at $a = \frac{\pi}{3}$ is given by

   $$f(x) = \sin x = \sum_{n=0}^{\infty} \frac{c_n \left(x - \frac{\pi}{3}\right)^n}{n!}.$$

3. (4pts) The series $\sum_n c_n 3^n$ converges and $\sum_n c_n(-3)^n$ diverges. If R is the radius of convergence of $\sum_n c_n(x-1)^n$, is it possible that $R < 3$? If **true**, explain why, if **false** give a counterexample.

   It is not possible, because the convergence of $\sum c_n 3^n$ implies that the power series converges at a distance of 3 away from the center.