

1. (3pts) Find the power series representing the function  $\frac{1}{1-x^2}$ .

$$\text{Since } \frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \frac{1}{1-x^2} = \sum_{n=0}^{\infty} x^{2n}.$$

2. (3pts) Find the Taylor series of the function  $e^x$  centered at  $a = 2$ .

Let  $f(x) = e^x$ . For each  $n$ ,  $f^{(n)}(x) = e^x$ . So,  $f^{(n)}(2) = e^2$ . Therefore the Taylor series of  $e^x$  centered at  $a = 2$  is given by

$$f(x) = e^x = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!} = \sum_{n=0}^{\infty} \frac{e^2(x-2)^n}{n!}.$$

3. (4pts) The series  $\sum_n c_n 2^n$  and  $\sum_n c_n (-3)^n$  converge. What is the minimal possible radius of convergence of  $\sum_n c_n x^n$ ?

Since the power series converges at distances of 2 and 3 away from the center, the radius of convergence must be at least 3.