1. (3pts) Determine (and justify) whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin n \cos n}{n^2}$$

The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges by the p-test. Since $0 \leq \frac{|\sin n \cos n|}{n^2} \leq \frac{1}{n^2}$, the given series converges absolutely by the comparison test. Therefore the given series converges.

2. (3pts) Determine (and justify) whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \left(-\frac{1}{3}\right)^n \left(\frac{n}{n+1}\right)^n$$

Applying the root test,

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \frac{1}{3} \cdot \frac{n}{n+1} = \frac{1}{3} < 1,$$

so the series converges.

3. (4pts) What's the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{2\pi(x+4)^n}{n^{1/3}}$$

Here, $c_n = \frac{2\pi}{n^{1/3}}$. Therefore

$$R = \lim_{n\to\infty} \left|\frac{c_n}{c_{n+1}}\right| = \lim_{n\to\infty} \frac{(n+1)^{1/3}}{n^{1/3}} = 1.$$

At $x = -3$, the series simplifies to $\sum_{n=1}^{\infty} \frac{2\pi}{n^{1/3}}$, which diverges by the p-test. At $x = -5$, the series becomes $\sum_{n=1}^{\infty} \frac{2\pi(-1)^n}{n^{1/3}}$, which converges by the alternating series test. So, the interval of converges of the power series is $[-5, -3)$.