1. (3pts) Determine (and justify) whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^{3/2}}$$

The series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges by the p-test. Since $0 \leq \frac{|\sin n|}{n^{3/2}} \leq \frac{1}{n^{3/2}}$, the given series converges absolutely by the comparison test. Therefore the given series converges.

2. (3pts) Determine (and justify) whether the series is convergent or divergent.

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}$$

Using the ratio test, we have

$$\begin{aligned} \lim_{n\to\infty} \left| \frac{\frac{(n+1)!}{(n+1)^{n+1}}}{\frac{n!}{n^n}} \right| &= \lim_{n\to\infty} \left| \frac{\frac{n!}{(n+1)^n}}{\frac{n!}{n^n}} \right| \\ &= \lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^{-n} \\ &= e^{-1} \\ &< 1. \end{aligned}$$

Therefore the series converges.

3. (4pts) What's the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{2^n (x-2)^n}{n^{1/3}}$$

Here, $c_n = \frac{2^n}{n^{1/3}}$. Therefore

$$R = \lim_{n\to\infty} \left| \frac{c_n}{c_{n+1}} \right| = \lim_{n\to\infty} \left| \frac{2^n (n+1)^{1/3}}{2^{n+1} n^{1/3}} \right| = \frac{1}{2}.$$

At $x = \frac{3}{2}$, the series simplifies to $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{1/3}}$ and converges by the alternating series test. At $x = \frac{5}{2}$, the series becomes $\sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ and diverges by the p-test. So the interval of convergence is $\left[\frac{3}{2}, \frac{5}{2}\right)$.