

1. (3pts) Is the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n + \sqrt{n}}$$

convergent or divergent? Justify your answer.

The sequence  $\frac{1}{n+\sqrt{n}}$  is positive, decreasing, and converges to zero. Therefore the given series converges by the alternating series test.

2. (3pts) Use the **Comparison Test** or **Limit Comparison Test** to determine whether the series converges or diverges. Verify your answer.

$$\sum_{n=1}^{\infty} \frac{\sin(1 + 2^{-n})}{2^n}$$

For all  $n$ ,  $0 < \sin(1 + 2^{-n}) \leq 1$ . Then  $0 < \frac{\sin(1+2^{-n})}{2^n} < \frac{1}{2^n}$ . Therefore the given series converges by the comparison test.

3. (4pts) Determine whether the statement true or false. If false give a counterexample.

- (a) (2pts) The series  $\sum_{n=1}^{\infty} a_n^2$  is divergent then  $\sum_{n=1}^{\infty} a_n^3$  is divergent.

False. Consider  $a_n = \frac{1}{\sqrt{n}}$ .

- (b) (2pts) The series  $\sum_{n=1}^{\infty} a_n$  and  $\sum_{n=1}^{\infty} b_n$  are convergent then the series  $\sum_{n=1}^{\infty} a_n + b_n$  is convergent.

True.