1. (3pts) Use the **Comparison Test** or **Limit Comparison Test** to determine whether the series converges or diverges. Verify your answer.

$$\sum_{n=1}^{\infty} \frac{\sin\left(\frac{1}{n}\right)}{n}$$

For all n, $0 < \sin\left(\frac{1}{n}\right) < \frac{1}{n}$. Therefore $0 < \frac{\sin\left(\frac{1}{n}\right)}{n} < \frac{1}{n^2}$. Then the given series converges by the comparison test.

2. (3pts) Is the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt[3]{n}\ln n}$$

convergent or divergent? Justify your answer.

The sequence $\frac{1}{\sqrt[3]{n}\ln n}$ is positive, decreasing, and approaches zero. Therefore the given series converges by the alternating series test.

3. (4pts) Determine whether the statement true or false. If false give a counterexample.

(a) (2pts) The series $\displaystyle\sum_{n=1}^{\infty} a_n$ is converegnt then $\displaystyle\sum_{n=1}^{\infty}(-1)^n a_n$ is convergent.

False. Consider $a_n = \frac{(-1)^n}{n}$.

(b) (2pts) The series $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent then the series $\displaystyle\sum_{n=1}^{\infty} a_n^2$ is convergent.

False. Consider $a_n = \frac{(-1)^n}{\sqrt{n}}$.